Exact solutions for the Gardner equation

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Abstract:

This talk is devoted to the solution of the extended quadratic-cubic Korteweg - de Vries (KdV) equation, also known as the Gardner equation. This equation is integrable by means of the Inverse Scattering Technique, and thus admits exact analytical solutions. Depending on the coefficients, solitons and breathers represent the nonlinear localized waves related to the discrete part of the spectrum of the associated scattering problem. The discrete spectrum, in contrast to the KdV case, may be complex, what results in existence of the breather solution, which is the generalization of the breather solution of the modified KdV equation. Due to two nonlinear terms in the evolution equations, specific limiting solitons exist, which are the algebraic soliton, algebraic breather and the wide (or "top-table") soliton. The composite nonlinear law supports the complicated manner of nonlinear wave interactions, which are studied in detail [1, 2]. These effects have been already confirmed partly within the frameworks of generalized equations. Generation of solitons and breathers from initial disturbances of simple shapes is studied by means of the solution of the associated scattering problem, as well as by the direct numerical integration of the evolution equation [3, 4]. Again, some of these results have been already reproduced in numerical simulations of extended and strongly nonlinear models, in particular, the generation of couples of solitary waves [5], and the formation of a breather in realistic conditions [6]. The higher-order generalization for the Gardner equation was presented in [7], and that model was shown to be close to the asymptotically integrable one. It is an intriguing fact that the Gardner equation describes exactly the velocity of nonlinear long interfacial waves [7, 8], what may explain the particular place of the Gardner equation in the description of interfacial and internal waves in a stratified fluid. The talk reviews the last author's research on the topic in collaboration with other researchers (see the reference list) and reports the recent results.

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